1. What is the smallest integer m such that $\dfrac{10!}{m}$ is a perfect square?

   (A) 2 (B) 7 (C) 14 (D) 21 (E) 35

2. Fermat determines that since his final exam counts as two tests, he only needs to score a 28 on it for his test average to be 70. If he gets a perfect 100 on the final exam, his average will be 88. What is the lowest score Fermat can receive on his final and still have an average of 80?

   (A) 60 (B) 66 (C) 68 (D) 70 (E) 72

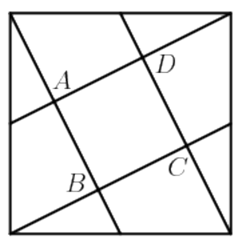
3. In the following base-10 equation, each of the letter represents a unique digit: $AM \cdot PM = ZZZ$. Find the sum of $A + M + P + Z$.

   (A) 15 (B) 17 (C) 19 (D) 20 (E) 21

4. If $|x| - x + y = 42$ and $x + |y| + y = 24$, then what is the value of $x + y$? Express your answer in simplest terms.

   (A) -4 (B) $26/5$ (C) 6 (D) 10 (E) 18

5. Lines from the vertices of a unit square are drawn to the midpoints of the sides as shown in the figure below. What is the area of quadrilateral $ABCD$? Express your answer in simplest terms.



   (A) $\dfrac{\sqrt{2}}{9}$ (B) $\dfrac{1}{4}$ (C) $\dfrac{\sqrt{3}}{9}$ (D) $\dfrac{\sqrt{8}}{8}$ (E) $\dfrac{1}{5}$

6. The operation # is defined by $x\#y = \dfrac{x-y}{xy}$. For how many real values a is $a\#(a\#2) = 1$?

   (A) 0 (B) 1 (C) 2 (D) 4 (E) infinitely many

7. How many unique 3-letter sequences with no spaces can be made using the letters in "AUGUSTIN LOUIS CAUCHY", which contains 19 letters? For example, "GAA" is one acceptable sequence, but "GGA" is not an acceptable sequence because there is only one G available. The original ordering of the letters does not have to be preserved.

   (A) 276 (B) 295 (C) 1486 (D) 1651 (E) 8086

8. A right rectangular prism is inscribed within a sphere. The total area of all the faces the prism is 88, and the total length of all its edges is 48. What is the surface area of the sphere?

   (A) 40π (B) $32\pi\sqrt{2}$ (C) 48π (D) $32\pi\sqrt{3}$ (E) 56π

9. How many ways are there to make change for 55 cents using any number of pennies, nickles, dimes, and quarters?

   (A) 42 (B) 49 (C) 55 (D) 60 (E) 78

10. The sum
$$\frac{1}{1+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{7}}+\cdots+\frac{1}{\sqrt{2n-1}+\sqrt{2n+1}}$$
is a root of the quadratic x^2+x+c. What is c in terms of n?

   (A) $-\frac{n}{2}$ (B) $2n$ (C) $-2n$ (D) $n+\frac{1}{2}$ (E) $n-2$

11. A group of 6 friends sit in the back row of an otherwise empty movie theater. Each row in the theater contains 8 seats. Euler and Gauss are best friends, so they must sit next to each other, with no empty seat between them. However, Lagrange called them names at lunch, so he cannot sit in an adjacent seat to either Euler or Gauss. In how many different ways can the 6 friends be seated in the back row?

   (A) 2520 (B) 3600 (C) 4080 (D) 5040 (E) 7200

12. Compute the remainder when $20^{(13^{14})}$ is divided by 11.

   (A) 1 (B) 3 (C) 4 (D) 5 (E) 9

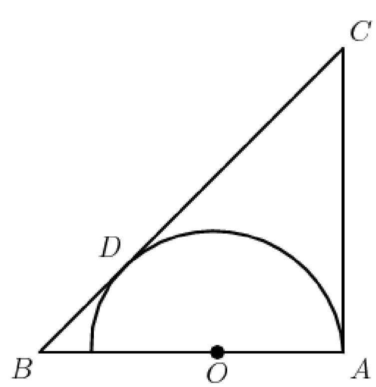
13. Four coplanar regular polygons share a common vertex but have no interior points in common. Each polygon is adjacent to two of the other polygons, and each pair of adjacent polygons has a common side length of 1. How many possible perimeters are there for all such configurations?

   (A) 2 (B) 3 (C) 4 (D) 5 (E) more than 5

14. Let $X=\{1,2,3,4\}$. Consider a function $f: X \to X$. Let $f^1=f$ and $f^{k+1}=(f \circ f^k)$ for $k \geq 1$. How many functions f satisfy $f^{2014}(x)=x$ for all x in X?

   (A) 9 (B) 10 (C) 12 (D) 15 (E) 18

15. Right triangle ABC has its right angle at A. A semicircle with center O is inscribed inside triangle ABC with the diameter along AB. Let D be the point where the semicircle is tangent to BC. If $AD=4$ and $CO=5$, find $\cos\angle ABC$.



   (A) $\frac{\sqrt{5}}{4}$ (B) $\frac{3}{5}$ (C) $\frac{12}{25}$ (D) $\frac{4}{5}$ (E) $\frac{2\sqrt{5}}{5}$

San Diego Math League High School Division, Round 2a
January 25, 2014

1. B
2. C
3. E
4. C
5. E
6. A
7. C
8. E
9. D
10. A
11. B
12. D
13. C
14. B
15. D

San Diego Math League High School Division, Round 2b
January 25, 2014

1. In base 10, the product 31×33 does not equal 1243. In what base does $31 \times 33 = 1243$?

2. A semicircle is joined to the side of a triangle, with the common edge removed. Sixteen points are arranged on the figure, as shown below. How many non-degenerate triangles can be drawn from the given points?

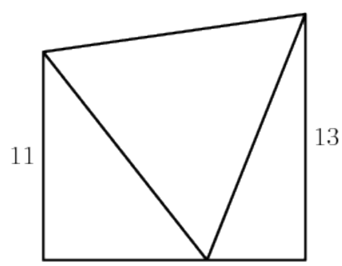
3. The 48 faces of 8 unit cubes are painted white. What is the smallest number of these faces that can be repainted black so that it becomes impossible to arrange the 8 unit cubes into a two by two by two cube, each of whose 6 faces is totally white?

4. $ABCD$ is a rectangle. Segment BA is extended through A to a point E. Let the intersection of EC and AD be point F. Suppose that measure of $\angle ACD$ is 60 degrees, and that the length of segment EF is twice the length of diagonal AC. What is the measure of $\angle ECD$?

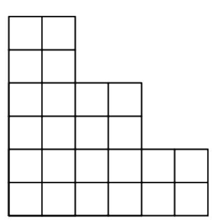
5. How many ways are there to make two 3-digit numbers m and n such that $n = 3m$ and each of six digits $1, 2, 3, 6, 7, 8$ are used exactly once?

6. The total number of edges in two regular polygons is 2014, and the total number of diagonals is 1,014,053. How many edges does the polygon with the smaller number edges have?

7. Two flag poles of height 11 and 13 are planted vertically in level ground, and an equilateral triangle is hung as shown in the figure so that lowest vertex just touches the ground. What is the length of the side of the equilateral triangle?



8. Twenty-four congruent squares are arranged as shown in the figure. In how many ways can we select 12 of the squares so that no two are diagonally adjacent? Directly adjacent spaces are acceptable.



San Diego Math League High School Division, Round 2b
January 25, 2014

1. 8
2. 540
3. 2
4. 20
5. 2
6. 952
7. 14
8. 112